Engineering Mechanics

Part 2: Dynamics

CHAPTER

8

L11

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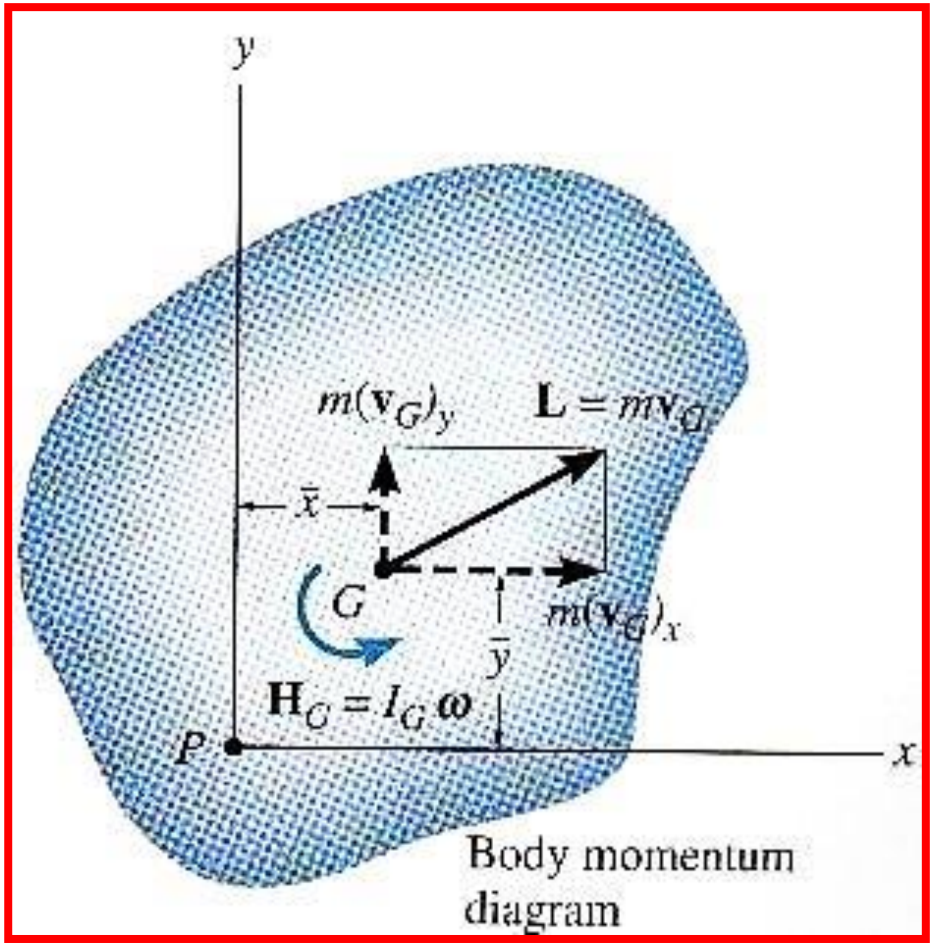
Kinetics of a Rigid Body

Planer Kinetics of a Rigid Body
Impulse and Momentum



Linear Momentum

$$\mathbf{L} = m\mathbf{v}_G$$

Angular Momentum

$$\mathbf{H}_G = I_G\,\boldsymbol{\omega}$$

Planer Kinetics of a Rigid Body
Impulse and Momentum

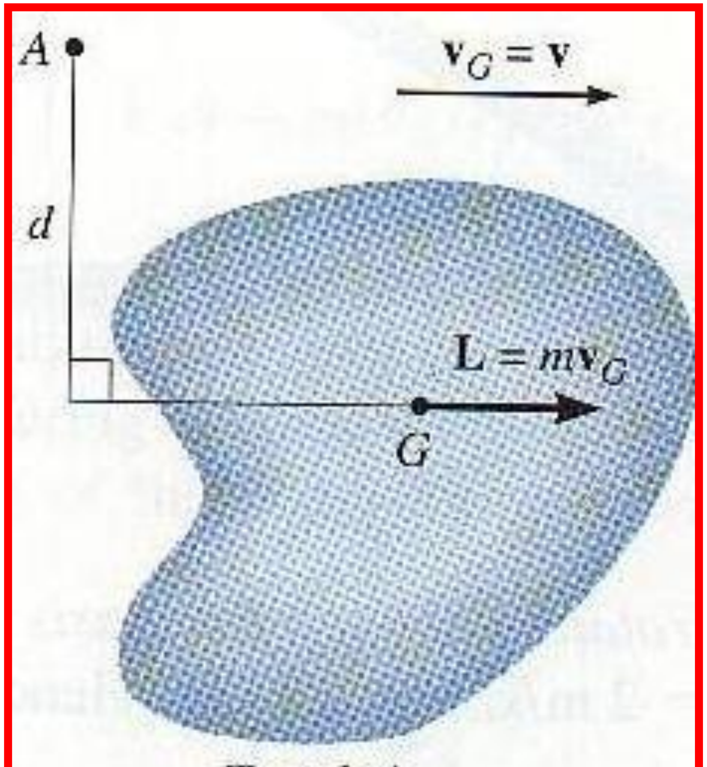
Translation

Translational motion (rectilinear or curvilinear)

$\boldsymbol{\omega} = \mathbf{0}$

$L = mv_G$, $H_G = 0$

$H_A = (d)(mv_G)$



Translation

Planer Kinetics of a Rigid Body
Impulse and Momentum

Rotation about a Fixed Axis

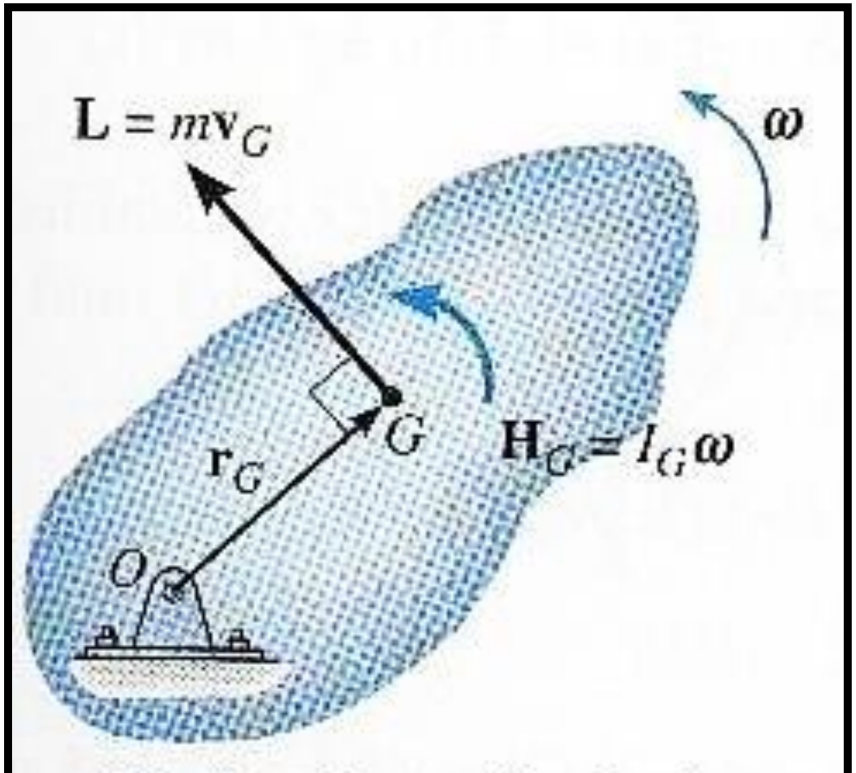
$L = m\,v_G$, $H_G = I_G\,\omega$

$H_O = I_G\omega + r_G(mv_G)$

since $v_G = r_G\omega$

$H_O = (I_G + mr_G^2)\,\omega$

$H_O = I_O\omega$



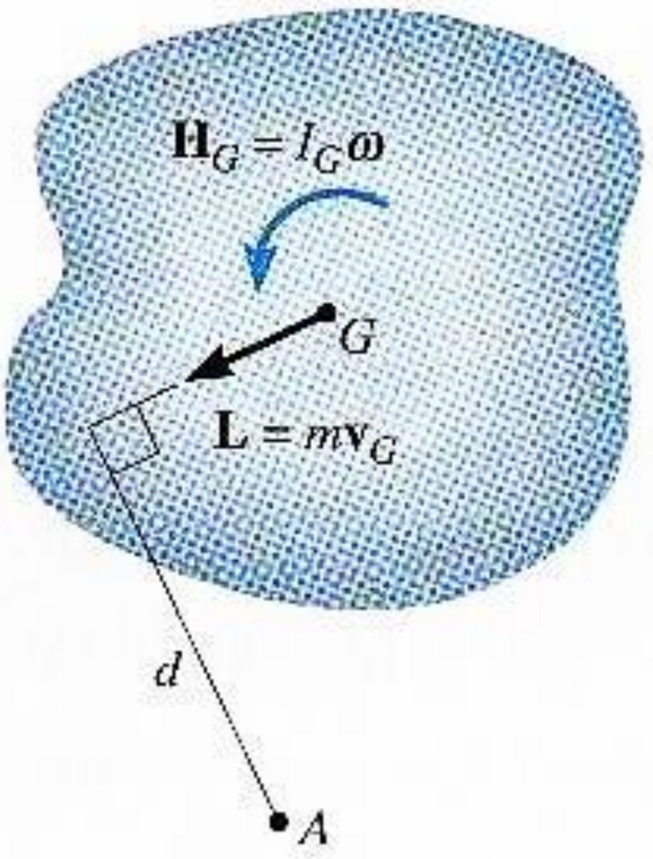
Rotation about a fixed axis

Planer Kinetics of a Rigid Body
Impulse and Momentum

General Plane Motion

$$L = mv_G \quad , H_G = I_G\,\omega$$

$$H_A = I_G\omega + (d)(mv_G)$$



General plane motion

Planer Kinetics of a Rigid Body
Impulse and Momentum

Principle of Linear Impulse and Momentum

$$m(\mathbf{v}_G)_1 + \sum \int_{t_1}^{t_2} \mathbf{F}\,dt = m(\mathbf{v}_G)_2$$

$(v_G)_1$ and $(v_G)_2$ are the velocity vectors of the center of mass at t_1 and t_2 respectively.

Scalar components for motion along x and y axes are:

$$m(v_{Gx})_1 + \sum \int_{t_1}^{t_2} F_x\,dt = m(v_{Gx})_2$$

$$m(v_{Gy})_1 + \sum \int_{t_1}^{t_2} F_y\,dt = m(v_{Gy})_2$$

Planer Kinetics of a Rigid Body
Impulse and Momentum

Principle of Angular Impulse and Momentum

$$I_O\omega_1 + \sum\int_{t_1}^{t_2} M_O\,dt = I_O\omega_2$$

Conservation of Linear Momentum

$$m(\mathbf{v}_G)_1 = m(\mathbf{v}_G)_2$$

Conservation of Angular Momentum

$$(I_G\omega)_1 = (I_G\,\omega)_2 \quad \textit{and} \quad (I_O\omega)_1 = (I_O\,\omega)_2$$

FUNDAMENTAL EQUATIONS OF DYNAMICS

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$v\,dv = a\,ds$	$v^2 = v_0^2 + 2a_c(s - s_0)$

FUNDAMENTAL EQUATIONS OF DYNAMICS

Particle Curvilinear Motion

x, y, z Coordinates

$v_x = \dot{x}$ $\quad a_x = \ddot{x}$

$v_y = \dot{y}$ $\quad a_y = \ddot{y}$

$v_z = \dot{z}$ $\quad a_z = \ddot{z}$

r, θ, z Coordinates

$v_r = \dot{r}$ $\quad a_r = \ddot{r} - r\dot{\theta}^2$

$v_\theta = r\dot{\theta}$ $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$v_z = \dot{z}$ $\quad a_z = \ddot{z}$

n, t, Coordinates

$v = \dot{s}$

$$a_t = \dot{v} = v\frac{dv}{ds}$$

$$a_n = \frac{v^2}{\rho} \qquad \rho = \left|\frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2}\right|$$

Relative Motion

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

FUNDAMENTAL EQUATIONS OF DYNAMICS

Rigid Body Motion About a Fixed Axis

Variable α	*Constant* $\alpha = \alpha_c$
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\, d\omega = \alpha\, d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \qquad v = \omega r \qquad a_t = \alpha r \qquad a_n = \omega^2 r$

FUNDAMENTAL EQUATIONS OF DYNAMICS

KINETICS

Mass Moment of Inertia $$I = \int r^2 \, dm$$

Parallel-Axis Theorem $$I = I_G + md^2$$

Radius of Gyration $$k = \sqrt{\frac{I}{m}}$$

FUNDAMENTAL EQUATIONS OF DYNAMICS

Equations of Motion

Particle	$\Sigma \mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ / $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

FUNDAMENTAL EQUATIONS OF DYNAMICS

Principle of Work and Energy

$$T_1 + U_{1-2} = T_2$$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F \cos\theta \, ds$
Constant force	$U_{F_c} = (F_c \cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

FUNDAMENTAL EQUATIONS OF DYNAMICS

Power and Efficiency

$$P = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v} \qquad \varepsilon = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_{\text{out}}}{U_{\text{in}}}$$

Conservation of Energy Theorem

$$T_1 + V_1 = T_2 + V_2$$

Potential Energy

$$V = V_g + V_e, \text{ where } V_g = \pm Wy,\ V_e = +\tfrac{1}{2}ks^2$$

FUNDAMENTAL EQUATIONS OF DYNAMICS

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma \int \mathbf{F}\, dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F}\, dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$$

Coefficient of Restitution

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

FUNDAMENTAL EQUATIONS OF DYNAMICS

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O \, dt = (\mathbf{H}_O)_2,$ where $H_O = (d)(mv)$
Rigid Body (Plane Motion)	$(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G \, dt = (\mathbf{H}_G)_2,$ where $H_G = I_G\omega$
	$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O \, dt = (\mathbf{H}_O)_2,$ where $H_O = I_G\omega + (d)(mv_G)$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$